Synthesis and Design of Wide-Band Equal-Ripple TEM Directional Couplers and Fixed Phase Shifters

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Abstract—An approximate synthesis technique is described for TEM directional couplers and fixed phase shifters consisting of multiple parallel-coupled quarter-wave sections. Synthesis is accomplished without the use of polynomials by a first-order relationship between the components and Chebyshev antenna arrays. The design procedure has the important advantage of allowing the maximum coupling coefficient between transmission lines to be an independent parameter. An iterative correction procedure for bringing the performance arbitrarily close to equal-ripple is described. The process has been programmed for electronic computation, and the results have been tabulated for directional couplers of 3, 6, 10, and 20 dB overall coupling and for phase shifters of 90, 45, and 22.5 degrees differential phase shifts. Bandwidth ratios range between 2 and 25, and the number of coupled sections ranges from 3 to 21 for couplers and from 2 to 9 sections for phase shifters. Maximum normalized even-mode impedance ranges from 1.83434 to 4.5.

List of Symbols

Z_{oe}	Even-mode characteristic impedance, one conductor to ground
Z_{oo}	Odd-mode characteristic impedance, one conductor to ground
Z_m	Maximum allowable even-mode impedance (normalized to $Z_0=1$)
Z_{lk}	Even-mode impedance of kth section in lth coupled region
ρ	Ratio of Z_{oe} to Z_{oo}
Γ_k	Reflection coefficient at junction of kth and $(k+1)$th sections
$\Gamma(\theta)$	Reflection coefficient as function of electrical length of section (frequency)
$\Gamma(\pi/2)$	Mid-band coupling of directional coupler. Coupling in dB $=20 \log \Gamma(\pi/2)$
θ	Electrical length of single coupled section
θ_0	Electrical length of single coupled section at low end of operating frequency band
θ_a	First value of θ for $T_n(a \cos \theta)=1$
θ_1, θ_2	First two zeros of $T_n(a \cos \theta)$
ϵ_r	Relative dielectric constant
Φ	Absolute phase through phase-shifter coupled regions
Ψ_0	Specified midband phase shift
u	$\Phi/2-M\theta$, where M is total number of coupled regions
$\Psi(\theta)$	Phase-shift response to be synthesized
N	Number of $\lambda/4$ sections in the largest cascaded device in the tandem solution
$T_n(a \cos \theta)$	Chebyshev polynomial of order n, argument $a \cos \theta$
$P(\theta)$	Integrated Chebyshev polynomial: $\int_0^\theta T_n(a \cos \theta) d\theta$
R	Ripple. Coupler ripple in dB $=20 \log R$. Phase-shifter ripple in degrees $=(180/\pi)(R-1)\Psi_0$
BW	Bandwidth $=f_2/f_1=(180-\theta_0)/\theta_0$
a_i	ith element of the Chebyshev antenna distribution; $a_1=1.0$
f_1, f_2	Lower and upper frequency limits of operating band
k_0	Voltage coupling coefficient, design goal
K	Scaling factor
α_0	Coupling design goal; $k_0=\sin \alpha_0$
Δ_i	ith relative maximum deviation from α_0, or Ψ_0
Δ_{av}	Average ripple in response function
δ_i	Desired amount of change at the ith relative maximum or minimum to make the response curve be equal ripple.

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Introduction

An important aspect of any multisection microwave network is the derivation of a general synthesis procedure. Generally, the potential value of the network is not fully realized until such a procedure is available. Many examples can be considered. The quarter-wave stepped impedance transformer and several types of resonator filters have been treated exhaustively in the literature [1]–[3].

Two closely related components are treated here. One, the multisection parallel-coupled TEM directional coupler, has been previously described [4]. Young emphasized the problem, stating the need for new insertion-loss functions [5].

The multisection TEM phase shifter has only recently received attention since Schiffman's initial efforts [6]. Cristal has described an exact synthesis of phase shifters, so that both components have received similar treatment [23].

The general concensus at this time seems to be that an iterative procedure is necessary in synthesizing these components. This type of approach is particularly attractive in view of the high-speed computing techniques now available. Workers at Stanford Research Institute and at Illinois Institute of Technology have treated the problem of deriving the polynomial for directional couplers on an iterative basis [7], [8]. The synthesis of the coupler is then a straightforward procedure. DuHamel has used dynamic programming to arrive at an equal-ripple coupler response without deriving the polynomial [9]. The particular method described in this paper, different from any previously described, has an important advantage in that the maximum coupling coefficient of the individual sections can be specified. Therefore, in addition to synthesizing components of theoretical interest, the engineer can design components of practical value since maximum coupling coefficient is generally a basic limitation of the transmission-line configuration being used. This approach yields a component with the fewest quarter-wave coupled sections for given performance.

Before the objectives are introduced, it is desirable to describe the prototype components that are being discussed. Both components are, in a sense, four-port networks. Both are ideally matched at all inputs. In one, the directional coupler, a change in the relative amplitude of the outputs in comparison to the inputs is achieved. In the other, the phase shifter, a change in the relative phase of the outputs in comparison to the inputs is achieved.

Figure 1 is a description of the basic multisection directional coupler. The various coupled sections are $\lambda/4$ in length at the center frequency. The contributions to the coupling curve are represented in terms of odd harmonics in θ, the electrical length of the coupled sections, which is proportional to frequency. If second-order effects are neglected, the harmonics can be attributed to specific steps, as indicated in Fig. 1(c).

The directional couplers considered here are symmetrical. The quadrature phase relationship between outputs of a symmetrical coupler is desirable in applications where close tolerances are to be held on phase [10]. Furthermore, the technique that will be described for realizing wide-band couplers with moderate coupling coefficients requires the quadrature phase characteristic. Thus, quadrature hybrid junctions in the form of 3-dB directional couplers are readily synthesized. Magic Tee hybrid junctions may be obtained by inserting a 90-degree phase shifter in the outputs of a 3-dB coupler [11].

Figure 2 is a description of the basic multisection phase shifter. The component consists of two separate transmission lines, one of which contains the coupled sections and the other of which is simply a length of uncoupled line. The effect of the coupled sections is to introduce dispersion, and the differential phase between lines is controlled by achieving the proper dispersive characteristics. It must be noted that the length of the uncoupled line is also a parameter. As for the coupler,

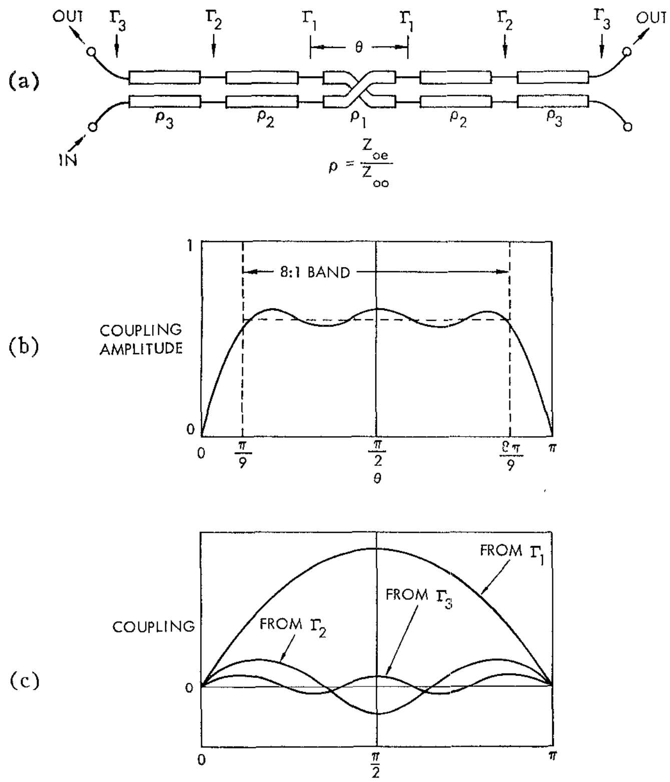


Fig. 1. Operation of multisection directional coupler. (a) Configuration of multisection directional coupler. (b) Required coupling response of multisection coupler. (c) Decomposition of coupling response.

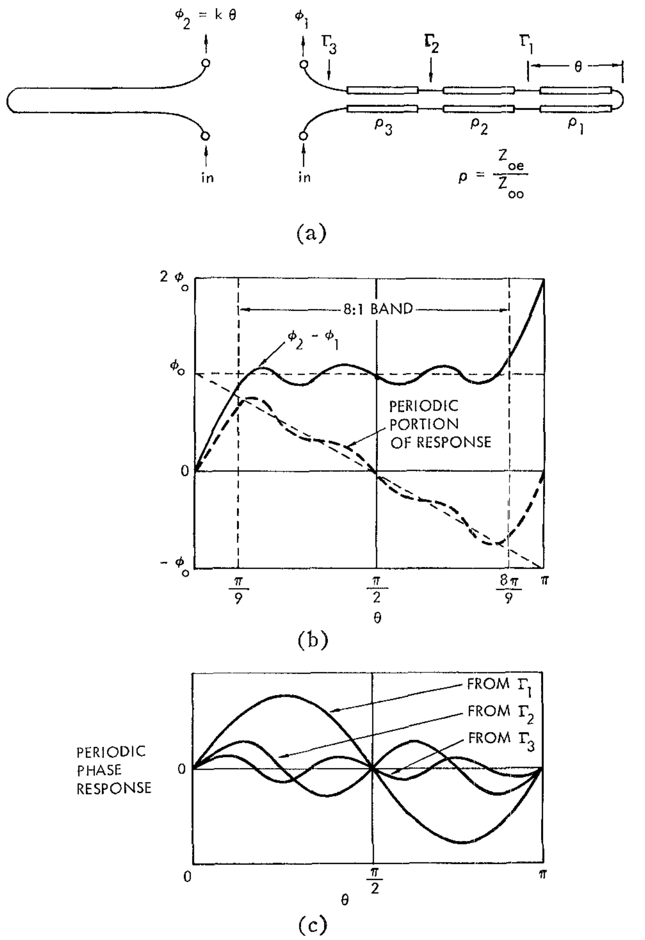


Fig. 2. Operation of multisection phase shifter. (a) Configuration of multisection phase shifter. (b) Required response of multisection phase shifter. (c) Decomposition of phase response.

the contributions to the dispersion curve are represented in terms of harmonics, even in this case, in θ. Again, if second-order effects are neglected, the harmonics can be attributed to specific steps.

The paper is divided into three parts. The first part describes an approximate synthesis procedure that enables the designer to specify bandwidth, overall coupling or phase shift, ripple, and maximum coupling coefficient.

The first part begins with a first-order analysis of the components, and the basis for an approximate harmonic synthesis is developed.

The actual synthesis procedure is derived in a section that is concerned first with the derivation of a suitable polynomial, then with the synthesis of the prototype component. It is found that the entire procedure can be performed, with the aid of Chebyshev antenna distribution tables, without actually writing down a polynomial.

A section is devoted to the practical aspects of achieving reasonable coupling coefficients and of realizing a satisfactory geometry for the resulting TEM transmission-line configuration. Reasonable coupling coefficients are made possible, regardless of the number of sections or bandwidth, by arranging coupled regions in series for the phase shifter and in tandem for the coupler.

The second part considers the correction of the design to obtain precise performance.

The correction procedure is based upon the derivation of an error curve from the calculated response of the initial approximate design. The error curve is harmonically analyzed to obtain corrections to the Γ_k, and the response of the modified component is calculated. This procedure is repeated until the calculated performance is satisfactorily close to equal-ripple.

A rather complete program has been generated for performing the precise design by digital computation. The inputs may be in the form of normalized or unnormalized reflection coefficients or even-mode impedances. The tolerance, or deviation from equal-ripple performance, may be specified. The response of the overall component, as well as the responses of the separate coupled regions, can be plotted.

Sample tables of computed designs are presented with bandwidth, number of sections, and maximum even-mode impedance as parameters.

A brief third part references experimental results obtained using the design techniques described here.

Approximate Synthesis Procedure

First-Order Analysis of Components

It has already been stated that the operation of the components under consideration can be described approximately in terms of first-order reflections of the even and odd modes. If the ratio of even- to odd-mode impedances for section j is given by $\rho_j = Z_{oej}/Z_{ooj}$, the even-mode reflection coefficient at the junction with the $j+1$ section is

$$\Gamma_j = \frac{\sqrt{\rho_{j+1}} - \sqrt{\rho_j}}{\sqrt{\rho_{j+1}} + \sqrt{\rho_j}} = \frac{Z_{oej+1} - Z_{oej}}{Z_{oej+1} + Z_{oej}}.$$

If the reflections from symmetrically located steps are combined, the result is

$$\Gamma_1 e^{j\theta} - \Gamma_1 e^{-j\theta} = 2j\Gamma_1 \sin\theta$$

where Z_1 is the central section, and θ is the electrical length of the section. The overall reflection coefficient for a multisection coupler is of the form,

$$\Gamma(\theta) = 2j[\Gamma_1 \sin\theta + \Gamma_2 \sin 3\theta + \Gamma_3 \sin 5\theta + \cdots]. \quad (1)$$

In (1), higher-order reflections have been neglected.

Some workers have approached the problem on the basis of finding the appropriate polynomial, since the synthesis of the impedances in terms of the polynomial is now available for both couplers and phase shifters. This approach was not used because its application would be limited by practical considerations. In the following paragraphs, emphasis is placed on approximate procedures wherever possible. The correction process involves feeding back the resulting errors to an early step in the design procedure. The philosophy here is to have the feedback "loop" include as much of the process as possible.

As in the case of the directional coupler, the performance of the phase shifter is defined in terms of the reflection coefficients of the even and odd modes, which are equal and of opposite sign for both cases. For the directional coupler, the amplitude of the reflection coefficient determines the coupling. For the phase shifter, however, the magnitude of the reflection coefficients is unity, and the phase of these coefficients determines the phase-shift characteristics of the component.

For the purposes of this part of the design, an approximate technique is adequate. The method described in the following paragraphs is comparable to the one used for the directional coupler in that it neglects higher-order reflections.

The basic coupled-section configuration for the phase shifter is similar to that of the directional coupler. The relationship among the various impedances is $Z_o = \sqrt{Z_{oo}Z_{oe}}$, and the operation of the component can be described in terms of only one of the coupled modes. The coupled region is terminated in such a way that the even mode is open-circuited and the odd mode is short-circuited. It is therefore found that the mode impedances have opposite signs, resulting in cancellation of the reflected waves at the input arm and reinforcement at the output, or coupled, arm.

The reflection coefficient of a stepped-impedance section of length θ, normalized impedance Z, terminated in short circuit, corresponding to the odd mode, is calculated as follows.

The input impedance is seen to be

$$Z_{in} = jZ \tan \theta.$$

The input reflection coefficient is

$$\Gamma_{in} = \frac{jZ \tan \theta - 1}{jZ \tan \theta + 1}.$$

It is found that the phase of the reflection coefficient, allowing for sign correction caused by the transfer of energy to the output arm of the component, is given by

$$\tan \frac{\phi}{2} = Z \tan \theta$$

where ϕ is the absolute phase through the component. The periodic term of the phase characteristic, shown in Fig. 2, is given by $u = \phi/2 - \theta$. After some manipulation, it is found that

$$\tan u = \frac{\left\{\frac{1-Z}{1+Z}\right\} \sin 2\theta}{1 + \left\{\frac{1-Z}{1+Z}\right\} \cos 2\theta},$$

and it is seen that the factor $(1-Z)/(1+Z)$ is just the magnitude of the reflection coefficient at the impedance step.

Thus, if the reflections and phase dispersion are small and second-order effects are neglected, the overall phase dispersion has the form,

$$u = \Gamma_{1,2} \sin 2\theta + \Gamma_{2,3} \sin 4\theta + \cdots$$

and the phase-shift response is $\Psi(\theta) = u + (\phi_0 \theta)/(\pi/2)$, where ϕ_0 is specified as the midband phase shift. On a first-order basis, the coupler and phase shifter have essentially equivalent representations. The coupler response consists of a series of odd sine harmonics, and the phase-shift response is made up of even sine harmonics.

It is at this point that the polynomial problem appears. Most components, such as step transformers from one impedance to another, or antenna arrays, or discrete directional couplers, can be analyzed in terms of sums of cosine harmonics. It is found that $\Sigma a_n \cos n\theta$ can be expressed as $\Sigma a_n \cos^n \theta$, and the Chebyshev polynomial can be applied. In the case of $\Sigma a_n \sin n\theta$, however, the equivalent power series is $\Sigma b_n \sin \theta \cos^{n-1} \theta$, and the remaining $\sin \theta$ factor causes difficulty.

Derivation of First-Order Reflection Coefficients

Ksienski described a technique for synthesizing sector-shaped antenna patterns, in which he integrated Chebyshev polynomials [12]. Price and Hyneman used both differentiation and integration of such polynomials to generate monopulse difference patterns [13].

The integration of even and odd Chebyshev polynomials, expressed in the form $T_n(a \cos \theta)$, is illustrated in Fig. 3. The similarity between the integrated patterns and the responses of the directional couplers and phase shifters is apparent. It is also seen that this step provides the proper mathematical form for both the couplers and the phase shifters. The polynomials of odd order provide coupler responses, and even order phase-shift responses.

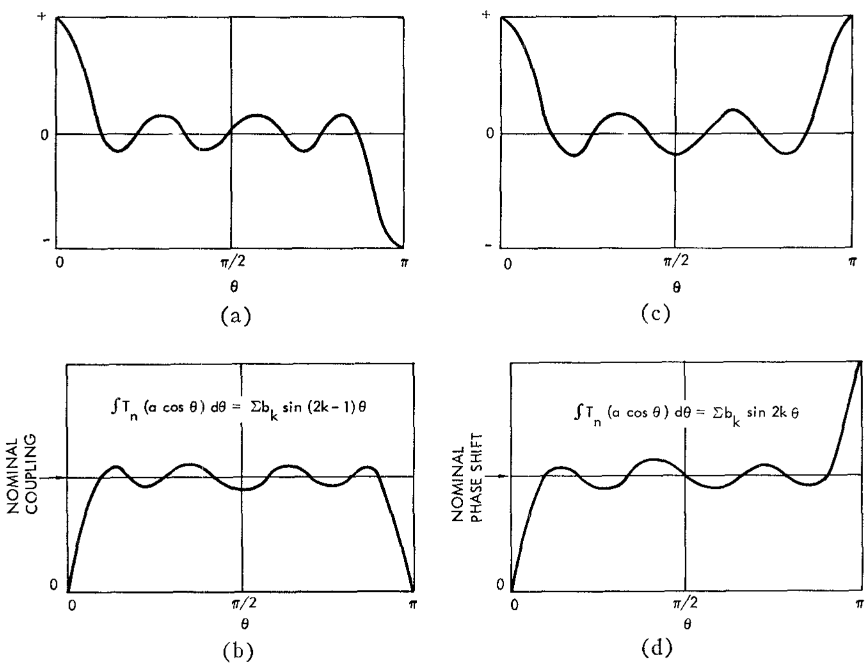


Fig. 3. Generation of polynomials for coupler and phase shifter. (a) Chebyshev polynomial $T_n(a \cos \theta)$—n odd. (b) Integral of odd polynomial. (c) Chevyshev polynomial $T_n(a \cos \theta)$—n even. (d) Integral of even polynomial.

Because the polynomial responses of the components have been related to antenna patterns, the possibility of relating the component reflection coefficients with the antenna array distribution is seen. The relationship is illustrated in Fig. 4. The antenna distribution and pattern are Fourier transforms, as are the reflection coefficients and response of the component. A basic result of Fourier analysis is that the integration of a function is accompanied by the division of its transform by the independent variable. For the phase shifter, with a reflection coefficient at the origin, the indeterminacy is disposed of by setting that reflection equal to one.

Therefore, if one can decide which Chebyshev antenna pattern to use, he can obtain the relative distribution of reflection coefficients by referring to the tables and appropriately operating upon them.

The polynomial T_n $(a \cos \theta)$ is specified by n and a. An alternative parameter is the sidelobe level of the corresponding antenna pattern, which is 20 log $T_n(a)$ in dB. In the design of the coupler or phase shifter the number of sections and required bandwidth are generally known. If ripple is specified rather than number of sections, it is necessary to try different n until the required ripple is achieved.

If the bandwidth and number of sections are known, the value of a or the sidelobe level must be determined. One approach to this problem is trial and error, whereby T_n $(a \cos \theta)$ is integrated with some arbitrary value of a chosen, the result examined for bandwidth, then a new value of a estimated, until the required bandwidth is obtained. Such a procedure, however, requires machine computation for high-order polynomials. Since the basic premise of the synthesis procedure to this point has been to obtain an approximate design, an approximate method for evaluating a is described.

Figure 5(a) is a plot of a polynomial T_n $(a \cos \theta)$, based on the following simplifying assumptions.

1) The curve is sinusoidal in shape between zeros, including the mainlobe region near $\theta = 0$.

2) The zeros are uniformly spaced in the sidelobe region. Figure 5(b) is a plot of the integral of the curve of Fig. 5(a), $P(\theta) = \int T_n(a \cos \theta) d\theta$. $P(\theta)$ is sinusoidal between maxima and minima. It is seen that the significant values of θ in Fig. 5(a) are θ_a, the first point where $T_n(a \cos \theta) = 1$, and θ_1 and θ_2, the first two zeros. In Fig. 5(b), θ_1 and θ_2 retain their significance because they are the first maximum and minimum of $P(\theta)$. The edge of the component's operating band, however, is defined by θ_0, where $P(\theta_0) = P(\theta_2)$. Referring back to the original polynomial curve $T_n(a \cos \theta)$, it is seen that the area between θ_0 and θ_1 is equal to the area between θ_1 and θ_2.

The following equations are now available for relating bandwidth to the parameter a, ripple, and number of sections:

$$\frac{R-1}{R} = \frac{\frac{\pi}{2} - \theta_1}{\left\{\frac{n-1}{2}\right\} T_n(a)\theta_1} \tag{2}$$

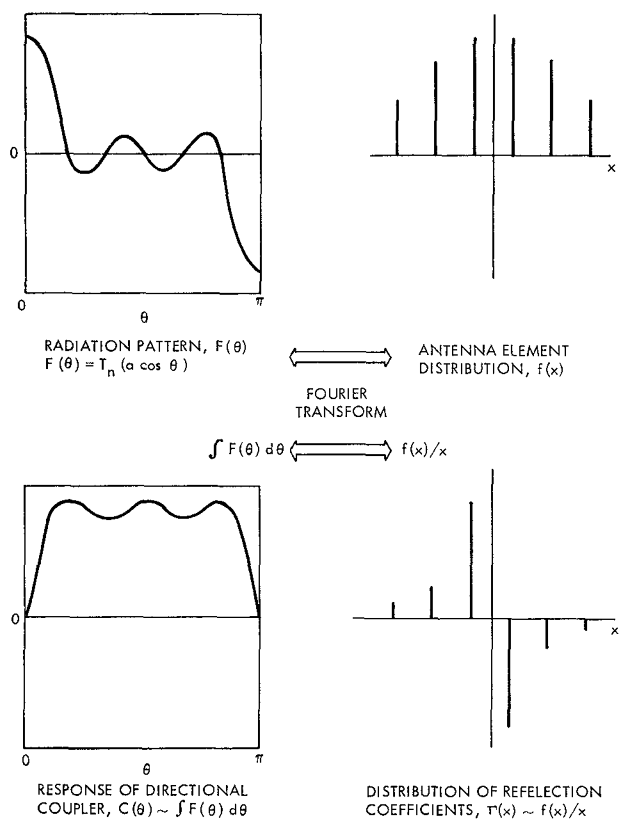


Fig. 4. Relationship between antenna arrays and components.

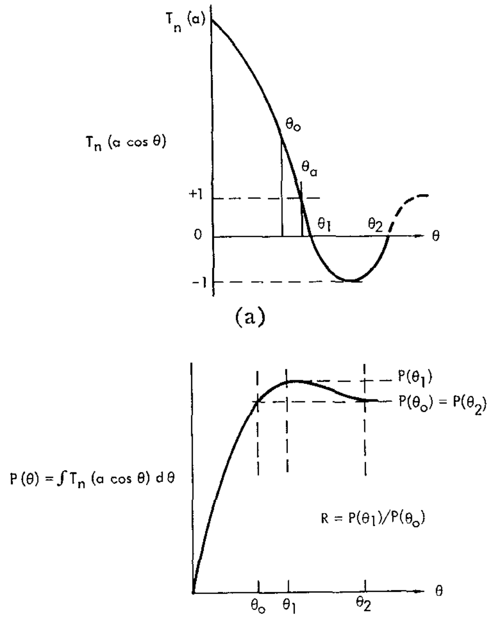


Fig. 5. Estimation of parameter a. (a) Chebyshev polynomial. (b) Integral of polynomial.

$$\log \left[T_n(a) + \sqrt{T_n^2(a) - 1}\right] = n \cosh^{-1} \left\{\frac{1}{\cos \theta_a}\right\} \quad (3)$$

$$\approx n\theta_a \qquad \text{for } \theta_a \text{ small}$$

$$\theta_a = \theta_1 \left\{1 - \frac{2}{\pi T_n(a)}\right\} \quad (4)$$

$$\sin \frac{\pi}{2} \frac{\theta_0}{\theta_1} = \frac{1}{R}\,. \quad (5)$$

Bandwidth is defined by

$$BW = \frac{f_2}{f_1} = \frac{\pi - \theta_0}{\theta_0},$$

where f_2 and f_1 are the upper and lower limits of the frequency band, respectively.

The derivation of (2) is based on the approximation that the Chebyshev polynomial is sinusoidal from zero to θ_1 and from θ_1 to θ_2. Equation (3) is an exact expression for $T_n(a)$ in terms of θ_a. Equation (4) is based on the approximation that the polynomial curve is linear for values of θ slightly less than θ_1. Equation (5) results from the sinusoidal shape of the integrated polynomial $P(\theta)$ between zero and θ_1.

Equations (2) through (5) contain two known parameters, θ_0 and n, and four unknown, θ_a, θ_1, $T_n(a)$, and R. Because the equations contain transcendental functions, an explicit solution is not readily available. An alternative technique is a judicious trial-and-error solution in which either $T_n(a)$ or θ_a is assigned a value. Then (3), (4), (2), and (5) are solved in order, resulting in a value of θ_0. The values of parameters are correct when the final value of θ_0 agrees with the specified value. To simplify the above procedure, Fig. 6 was prepared. It shows $20\log_{10}\{T_n(a)\}$ versus bandwidth for $n = 3, 5, 7, \cdots, 21$.

When $T_n(a)$ has been determined, it is possible to find the corresponding element amplitude distribution in the Chebyshev antenna tables [14]. This distribution can be used to find the reflection coefficients of the component in the following manner.

Assume that the element amplitude distributions for a coupler and phase shifter are shown in Table I. The objective in Table I is to progress from the antenna distribution found in the Chebyshev antenna tables to the even-mode impedances of the various coupled regions. The first step is to obtain relative reflection co-

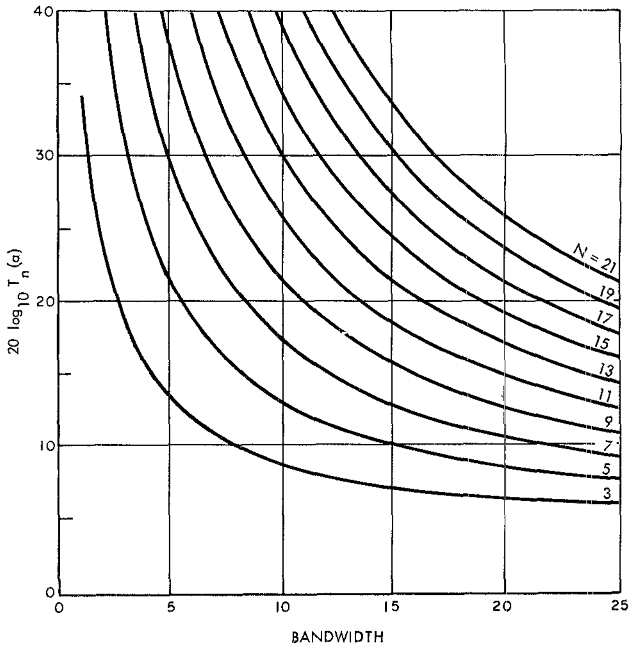


Fig. 6. Bandwidth vs. 20 $\log_{10} T_n(a)$ as a function of N, number of coupler sections.

TABLE I
DETERMINATION OF Z_k

Characteristics	Coupler ($n = 5$)	Phase Shifter ($n = 4$)
Element amplitudes	$a_3,\ a_2,\ a_1$	$a_3,\ a_2,\ a_1$
Relative reflection coefficient	$b_3 = \frac{a_3}{5};\ \ b_2 = \frac{a_2}{3};\ \ b_1 = a_1$	$b_3 = \frac{a_3}{3};\ \ b_2 = \frac{a_2}{2};\ \ b_1 = a_1$
Coupling at $\theta = \pi/2$	$B = b_1 - b_2 + b_3$	
Phase shift at $\theta = \pi/4$		$B = b_1 - b_2 + b_3$
Scaling factor	$K = \frac{\Gamma(\pi/2)}{B}$	$K = \frac{u(\pi/4)}{B} \approx \frac{\psi_0}{2B}$
Reflection coefficients	$\Gamma_3 = \frac{Kb_3}{2},\ \ \Gamma_2 = \frac{Kb_2}{2},\ \ \Gamma_1 = \frac{Kb_1}{2}$	$\Gamma_3 = Kb_3;\ \ \Gamma_2 = Kb_2;\ \ \Gamma_1 = Kb_1;\ \ \Gamma_0 = 1$
Even-mode impedances ($Z_0 = 1$)	$Z_3 = \frac{1 + \Gamma_3}{1 - \Gamma_3}$	$Z_3 = \frac{1 + \Gamma_3}{1 - \Gamma_3}$
	$Z_2 = Z_3 \frac{1 + \Gamma_2}{1 - \Gamma_2}$	$Z_2 = Z_3 \frac{1 + \Gamma_3}{1 - \Gamma_3}$
	$Z_1 = Z_2 \frac{1 + \Gamma_1}{1 - \Gamma_1}$	$Z_1 = Z_2 \frac{1 + \Gamma_3}{1 - \Gamma_3}$

efficients by dividing the amplitudes by 1, 3, 5 · · · for the coupler and by 1, 2, 3 · · · for the phase shifter. Next, it is necessary to find a scaling factor that adjusts the coefficients for the required coupling or phase-shift response. The parameter B results from setting $\theta = \pi/2$ for the coupler or $\theta = \pi/4$ for the phase shifter. Once the actual reflection coefficients have been determined, the even-mode impedances are found by working in from the uncoupled inputs to the component.

The drawback to this procedure is the high values of Z_1, which are found for wide-band multisection components with strong coupling or large phase shift. These high values represent extremely tight coupling, which may not be realizable in a given practical configuration. Admittedly, advances have been made in the coupling achievable between two transmission lines [15], so that the ratio of even- to odd-mode impedances can be approximately twenty [16]. However, it is characteristic of these components that, for any given overall coupling level, ripple, and maximum value of $\rho = Z_{oe}/Z_{oo}$, it is possible to specify a bandwidth sufficiently wide that the given parameters cannot be held.

A method for specifying maximum coupling coefficient as one of the design parameters is described in the next section.

Tandem Configuration for Practical Design

The only possible solution to the problem of achieving a design with specified maximum coupling coefficient (short of using a basically different cnfiguration such as that proposed by DuHamel, for example) is to split the coupling or phase shift into pieces and then to realize the required network for each piece [17].

Possible design techniques for the directional coupler are illustrated in Fig. 7. The analogous configuration for the phase shifter is easily visualized. The basis for the approach is the connection of coupled regions in tandem, the coupled and straight-through outputs of one multisection coupler being used as the isolated inputs to another coupler. The operation of such a network is mathematically equivalent to multiple discrete coupling in branchline or waveguide multihole couplers. The quadrature phase characteristic of symmetrical couplers is required. It is seen that the coupled lines must cross over at the center of each coupler. The important aspect of this configuration is the representation of the coupling coefficient of each region by the sine of an angle, which allows one to find the overall coupling level by adding angles.

It is interesting to note that, although this technique was assumed to be original, it has been learned that Monteath proposed the "cascade" connection of symmetrical directional couplers and suggested that wideband performance could be obtained by using multiple sections in each coupler [18].

Two tandem arrangements of coupled regions are shown in Figs. 7(b) and (c). In one, the separate regions

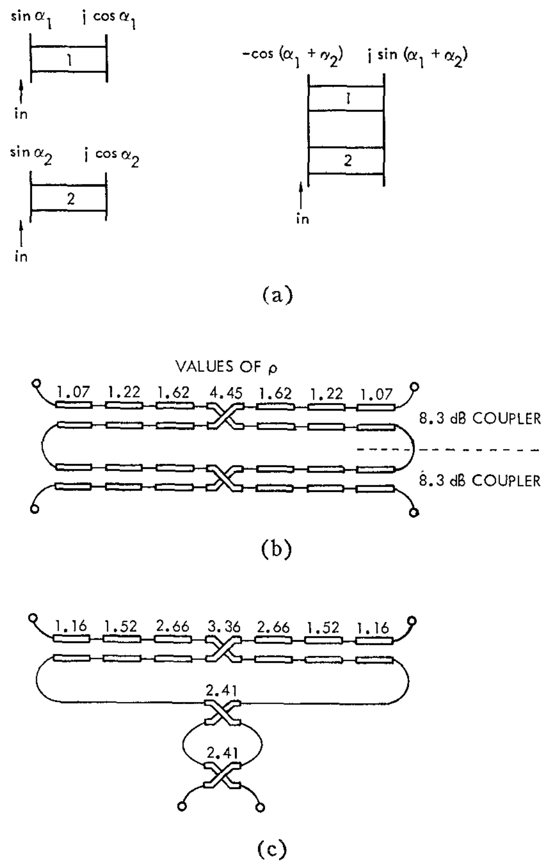


Fig. 7. Layout of directional couplers without extremely tight coupling. (a) Operation of directional couplers connected in series. (b) Series connection of two identical units. (c) Series connection of dissimilar units.

are identical. For example, a -3 dB coupler is realized by using two -8.3 dB couplers in tandem. (-3 dB$=20 \log \sin \pi/4$, and -8.3 dB$=20 \log \sin \pi/8$.) In general, the second arrangement is preferred because it requires the minimum number of total sections for specified maximum coupling coefficient. Furthermore, the minimum coefficient is greater than for the case of identical coupled regions. Too low a coupling coefficient, as well as one too high, presents practical difficulties.

The final step in the approximate synthesis procedure can now be defined. A maximum value of even-mode impedance is specified and represented by Z_m. The case of interest is the one for which at least one of the Z_k found sequentially by the procedure of Table I exceeds Z_m. The rule that holds in this case is that all Z_k initially greater than Z_m are set equal to Z_m. The resulting multisection coupled region satisfies the requirements for some of the Γ_k, in particular, those corresponding to the higher harmonics of the response function. Therefore, a new set of Γ_k is found for the second multisection coupled region, which will be placed in tandem with the first, by subtracting the actual Γ_k of the first region from the required values. This procedure is repeated until the totals of the corresponding Γ_k of the tandem regions equal the required Γ_k.

An example will clarify the procedure: Assume

$$Z_m = 1.8343.$$ [1]

A 3-dB directional coupler with 17:1 bandwidth ($\theta_0=\pi/18$) and 13 sections is being synthesized; the correct Γ_k are:

	Γ_7	Γ_6	Γ_5	Γ_4	Γ_3	Γ_2	Γ_1
	0.0303	0.0277	0.0403	0.0575	0.0881	0.1536	0.4989
Thus							
Values of Z_k, 1st Region	1.0799	1.1362	1.2269	1.3780	1.6539	1.8343	1.8343
Remaining Γ_k						0.0944	0.4989
Values of Z_k, 2nd Region						1.2089	1.8343
Remaining Γ_k							0.293
Values of Z_k, 3rd Region							1.8303

In the early application of this correction technique, only step 1) was performed by machine computation [19]. Manual computation of the remaining three steps was laborious. Such factors as number of sections, bandwidth, and maximum coupling coefficient, and number of iterations were limited by this procedure. Inevitable human errors introduced "noise" with a divergent effect on the iterative process. Since all steps can be specified logically, it is reasonable to attempt to program the entire design for machine computation.

The results of this approximate synthesis procedure can be expressed as a set of Z_{lk}, where the subscript l refers to the tandem region and the subscript k refers to the section within the tandem region.

The procedure described in the preceding sections is based on a first-order description of the components, and as such produces performance errors to the extent of the higher-order contributions. In the second part of this paper, a method is described for correcting the design to any required accuracy by successive iterations.

Precise Design and Tabulated Results

Correction Procedure

The first step in the precise design is to calculate the performance of the coupler or phase shifter that has just been synthesized by the previously described methods. This is a straightforward process and need not be discussed.

The calculated performance of the component will not, in general, exhibit the desired equal-ripple characteristic for two reasons. First, the design polynomial is the integral of an equal-ripple Chebyshev function and is not precisely equal-ripple. Second, the neglected higher-order effects will further degrade the calculated performance.

The objective of the correction procedure is to perturb the values of Γ so that the performance of the component is more nearly equal-ripple. This is done by the following steps:

1) the ripple of the desired curve is estimated from the initial performance curve;
2) an error curve is obtained by subtracting the initial curve from the desired curve,
3) a set of $\Delta\Gamma_k$ are calculated from the error curve by a Fourier analysis, and the original Γ_k are accordingly modified;
4) the impedance values of the first iteration are calculated from the new Γ_k.

[1] This value of Z_m results from a parallel-coupled, strip-transmission-line configuration in which $s/b=1/5$, $Z_0=50$ ohms, and $\epsilon_r=2.32$ (polyolefin dielectric).

The estimated ripple is found from

$$\Delta_{av} = \frac{\left(\sum_{i=0}^{n-1} 2\Delta_i\right) + \Delta_n}{2n+1}$$

for directional couplers and from

$$\Delta_{av} = \frac{\sum_{i=0}^{n} \Delta_i}{n+1}$$

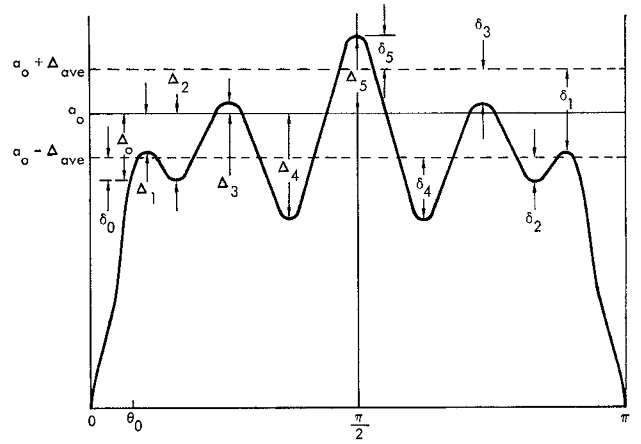
for phase shifters, where Δ_i is the ith relative maximum deviation of the calculated performance curve from the design objective, α_0. The value of Δ_0 is measured at the band edge, $\theta=\theta_0$.

The error curve is obtained by taking the differences between Δ_{av} and the Δ_i. Figure 8 is an illustration of the derivation of the error curve for a directional coupler.

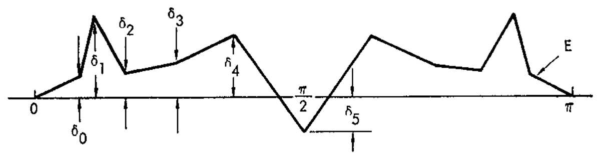
A Fourier analysis of the error curve will yield corrections for the initial reflection coefficients. Specifically, the correction for the ith reflection coefficient is $\Delta\Gamma_i = b_i/(2\pi)$ for couplers and $\Delta\Gamma_i = b_i/\pi$ for phase shifters, where b_i is the ith Fourier coefficient found from the error curve. With the improved reflection coefficients the whole process can be repeated until the desired solution is found. Figure 9 shows the initial response curve[2] and the results of the first two iterations. Most designs do not require more than about five iterations.

Although this simple correction procedure utilizes information gathered at the relative maxima and minima and the initial response curve does not always have the correct number of maxima and minima, the following computer program has solved all the given design problems. Obviously, the better the initial design, the fewer iterations are required.

[2] The initial response curve was obtained by using a 20-dB Chebyshev antenna distribution as indicated by Fig. 6.



(a)



(b)

Fig. 8. Derivation of error curve from computed response. (a) Unequal ripple coupler response showing error curve components. (b) Construction of error curve by straight line segments.

Program for the Correction Procedure

A simplified flow chart of the computer program used in this work is shown in Fig. 10.[3] Most of the blocks are self-explanatory. It should be noted that the input data to the program can be ir terms of normalized even-mode impedances Z_{oe}, reflection coefficients Γ, or antenna element distribution as in the example of Fig. 9. Also, whenever a component consisting of tandem connected coupled regions is to be designed, the program will provide the coupling or phase-shift response of the individual coupled regions. This feature can be useful in development work, where one might want to optimize each coupler before actually connecting all in tandem.

Because this design technique uses approximate methods, this computer program is not "optimum." However, it is of engineering value because designs of arbitrarily great precision can be efficiently achieved. In the previous sample computation, each iteration required less than one second of machine execution time. Furthermore, designs of greater difficulty involving more sections, larger bandwidths, and the like, require only slightly more machine time, with the result that the parameters of any practical component of this type can be obtained by this program, at least within the foreseeable future.

[3] Persons wishing to use this program are referred to: J. A. Mosko, "Finding the optimum designs of arbitrary TEM quadrature couplers and phase shifters, using a digital computer," U. S. Naval Ordnance Test Station, China Lake, Calif., NAVWEPS Rept. 9048, September 1965. This report, complete with sample data cards, contains a FORTRAN IV program listing and flow chart of the main program and all the subroutines (for plotted outputs, etc.) as it was used with the IBM 7094 at NOTS to generate Tables A through , (see footnote 7).

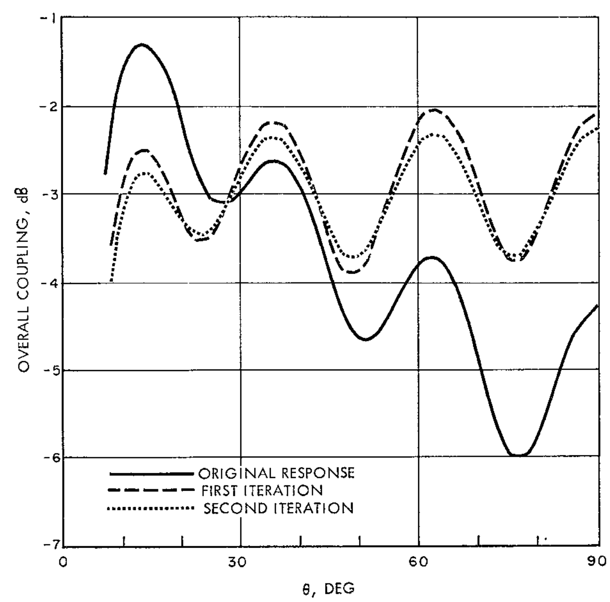


Fig. 9. Initial response and first two iterations—3-dB coupler with 13 sections, 17:1 bandwidth.

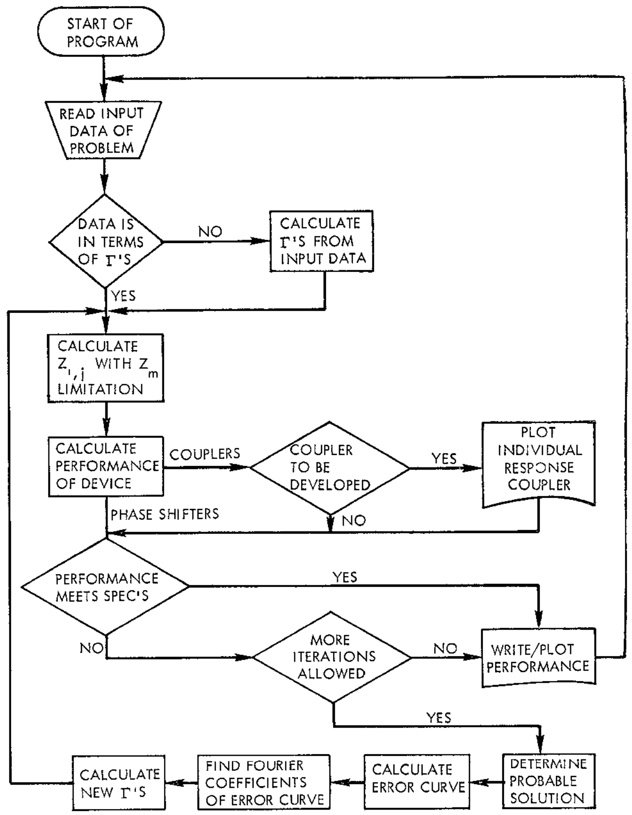


Fig. 10. Simple flow chart of computer program

Computed Results

Extensive coupler and phase-shifter designs have been tabulated. Tables II and III are examples of the twenty-two pages of tables that are available through the Auxiliary Publications Program of the American Documentation Institute.[4]

[4] A description appears in R. W. Zimmerer, "ADI auxiliary publications program," *IEEE Trans. on Microwave Theory and Techniques* (*Correspondence*), vol. MTT-13, pp. 883–884, November 1965.

TABLE II

EVEN-MODE IMPEDANCES FOR −3 dB COUPLER OF 13 SECTIONS

N=13									COUPLING=	-3.01 DB
BANDWIDTH	10.000	10.000	10.000	10.000	10.000	12.000	12.000	12.000	12.000	12.000
RIPPLE, DB	0.06248	0.06087	0.05990	0.05323	0.04458	0.12103	0.11704	0.11515	0.10373	0.08850
TOLERANCE *	0.00016	0.00022	0.00027	0.00022	0.00019	0.00049	0.00056	0.00045	0.00052	0.00035
Z(1, 1)	1.83434	2.09434	2.30314	3.30000	4.50000	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 2)	1.83434	1.96564	1.98223	2.00302	2.00300	1.83434	2.05199	2.07010	2.09084	2.10225
Z(1, 3)	1.44480	1.46136	1.46528	1.46508	1.44732	1.50055	1.52319	1.52789	1.52881	1.51232
Z(1, 4)	1.22892	1.23428	1.23492	1.23133	1.22036	1.27039	1.27919	1.28006	1.27645	1.26502
Z(1, 5)	1.11387	1.11525	1.11519	1.11246	1.10664	1.14395	1.14641	1.14639	1.14329	1.13677
Z(1, 6)	1.05092	1.05109	1.05092	1.04942	1.04673	1.07050	1.07080	1.07063	1.06875	1.06548
Z(1, 7)	1.01808	1.01792	1.01776	1.01704	1.01611	1.02883	1.02850	1.02825	1.02710	1.02581
Z(2, 1)	1.83434	1.60495	1.53375	1.67317	1.25386	1.83434	1.64331	1.57025	1.75050	1.30978
Z(2, 2)	1.05750	......	......	......	......	1.10535	......	......	......	......
Z(3, 1)	1.58882	1.60495	1.53375	......	......	1.66293	1.64331	1.57025	......	......
BANDWIDTH	14.000	14.000	14.000	14.000	14.000	17.000	17.000	17.000	17.000	17.000
RIPPLE, DB	0.19574	0.18846	0.18491	0.16838	0.14572	0.33259	0.32183	0.31322	0.28820	0.25346
TOLERANCE *	0.00078	0.00091	0.00084	0.00074	0.00058	0.00138	0.00136	0.00102	0.00105	0.00062
Z(1, 1)	1.83434	2.09434	2.30314	3.30000	4.50000	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 2)	1.83434	2.09434	2.15074	2.17078	2.19296	1.83434	2.09434	2.26225	2.27889	2.31652
Z(1, 3)	1.55397	1.57604	1.58600	1.58828	1.57313	1.62971	1.64498	1.66732	1.67094	1.65823
Z(1, 4)	1.30991	1.32077	1.32303	1.31972	1.30793	1.36591	1.37788	1.38453	1.38138	1.36945
Z(1, 5)	1.17366	1.17693	1.17721	1.17404	1.16684	1.21737	1.22136	1.22285	1.21933	1.21147
Z(1, 6)	1.09106	1.09146	1.09122	1.08915	1.08529	1.12310	1.12357	1.12335	1.12087	1.11635
Z(1, 7)	1.04134	1.04073	1.04025	1.03878	1.03705	1.06254	1.06175	1.06111	1.05862	1.05620
Z(2, 1)	1.83434	2.09434	1.60337	1.82255	1.36187	1.83434	2.09434	1.64882	1.92339	1.43466
Z(2, 2)	1.14918	1.00565	......	......	......	1.20886	1.06379	......	......	......
Z(3, 1)	1.73232	1.34337	1.60337	......	......	1.83025	1.41702	1.64882	......	......
BANDWIDTH	20.000	20.000	20.000	20.000	20.000	25.000	25.000	25.000	25.000	25.000
RIPPLE, DB	0.48866	0.47278	0.45866	0.42302	0.37751	0.76043	0.74186	0.72152	0.66213	0.59883
TOLERANCE *	0.00184	0.00220	0.00169	0.00125	0.00157	0.00245	0.00333	0.00294	0.00179	0.00254
Z(1, 1)	1.83434	2.09434	2.30314	3.30000	4.50000	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 2)	1.83434	2.09434	2.30314	2.37805	2.43102	1.83434	2.09434	2.30314	2.52880	2.60629
Z(1, 3)	1.70450	1.71217	1.73474	1.74832	1.73840	1.82149	1.82215	1.83444	1.86900	1.86412
Z(1, 4)	1.41841	1.43153	1.44070	1.44016	1.42837	1.50145	1.51579	1.52617	1.53345	1.52226
Z(1, 5)	1.25973	1.26421	1.26674	1.26368	1.25548	1.32700	1.33328	1.33625	1.33536	1.32658
Z(1, 6)	1.15591	1.15593	1.15579	1.15320	1.14843	1.20997	1.21023	1.20971	1.20646	1.20101
Z(1, 7)	1.08620	1.08455	1.08342	1.08006	1.07727	1.12707	1.12504	1.12314	1.11705	1.11296
Z(2, 1)	1.83434	2.09434	2.30314	2.02064	1.50501	1.83434	2.09434	2.30314	2.17799	1.61824
Z(2, 2)	1.28595	1.11770	1.00337	......	......	1.38040	1.20185	1.08523	......	......
Z(3, 1)	1.38759	1.48808	1.23996	......	......	1.44268	1.60639	1.33574	......	......
Z(4, 1)	1.36759	......	......	......	......	1.44268	......	......	......	......

TABLE III
EVEN-MODE IMPEDANCES FOR 90° PHASE SHIFTER OF 9 SECTIONS

N= 9 PHASE SHIFT= 90.0 DEG

BANDWIDTH	14.000	14.000	14.000	14.000	14.000	17.000	17.000	17.000	17.000	17.000
RIPPLE, DEG	1.24650	1.25441	1.47138	1.90465	2.21774	2.56785	2.73168	2.87317	3.72570	4.31304
TOLERANCE *	0.01086	0.01051	0.01159	0.01142	0.02106	0.01519	0.00716	0.02714	0.02228	0.02126
REF. LENGTH	7.75000	7.25000	6.75000	5.75000	5.25000	8.25000	7.75000	6.75000	6.25000	5.75000
Z(1, 1)	1.83434	2.09434	2.30314	3.30000	4.50000	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 2)	1.83434	2.09434	2.30314	3.30000	4.06562	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 3)	1.83434	2.09434	2.30314	2.43362	2.45644	1.83434	2.09434	2.30314	2.79781	2.63875
Z(1, 4)	1.72473	1.80331	1.75800	1.80610	2.00027	1.83434	1.97276	2.01425	1.92580	2.20304
Z(1, 5)	1.40164	1.39619	1.39159	1.54597	1.60486	1.52085	1.53998	1.52339	1.65822	1.74836
Z(1, 6)	1.20826	1.22909	1.26711	1.34023	1.38634	1.29998	1.29686	1.32114	1.45630	1.50813
Z(1, 7)	1.11675	1.15740	1.17736	1.20248	1.23056	1.16401	1.19587	1.23242	1.27980	1.32190
Z(1, 8)	1.07294	1.09804	1.09360	1.11475	1.13110	1.10040	1.13888	1.15793	1.17388	1.20000
Z(1, 9)	1.03814	1.04435	1.04239	1.05640	1.06503	1.06167	1.07902	1.08408	1.09991	1.11282
Z(2, 1)	1.83434	2.09434	2.30314	3.01178	2.18153	1.83434	2.09434	2.30314	3.30000	2.58474
Z(2, 2)	1.83434	1.98403	1.82191	1.33341	......	1.83434	2.09434	2.09979	1.55301	1.02914
Z(2, 3)	1.26580	1.11216	1.04427	......	......	1.42113	1.24424	1.14847	......	......
Z(2, 4)	......	......	......	......	......	1.03574	......	......	......	......
Z(3, 1)	1.83434	2.09434	1.89549	......	......	1.83434	2.09434	2.26020	1.01884	......
Z(3, 2)	1.19417	......	......	......	......	1.35352	1.06151	......	......	......
Z(4, 1)	1.57248	1.13801	......	......	......	1.78290	1.23753	......	......	......

BANDWIDTH	20.000	20.000	20.000	20.000	20.000	25.000	25.000	25.000	25.000	25.000
RIPPLE, DEG	3.98701	4.63562	4.77476	6.01895	6.86897	6.71283	8.07736	8.70456	10.39438	11.60516
TOLERANCE *	0.01472	0.05700	0.02247	0.06106	0.06226	0.08835	0.10035	0.13101	0.04112	0.09207
REF. LENGTH	8.75000	8.25000	7.75000	6.25000	5.75000	9.25000	8.25000	8.25000	6.75000	5.75000
Z(1, 1)	1.83434	2.09434	2.30314	3.30000	4.50000	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 2)	1.83434	2.09434	2.30314	3.30000	4.50000	1.83434	2.09434	2.30314	3.30000	4.50000
Z(1, 3)	1.83434	2.09434	2.30314	3.12927	2.93705	1.83434	2.09434	2.30314	3.30000	3.47361
Z(1, 4)	1.83434	2.09434	2.22492	2.06677	2.31778	1.83434	2.09434	2.30314	2.42168	2.51195
Z(1, 5)	1.64760	1.68041	1.67029	1.74774	1.90297	1.83434	1.91669	1.93658	1.91494	2.13217
Z(1, 6)	1.41122	1.38851	1.38812	1.57458	1.61604	1.58010	1.58962	1.56880	1.70687	1.81658
Z(1, 7)	1.24095	1.23989	1.27682	1.36705	1.41717	1.40410	1.36039	1.36868	1.52544	1.57891
Z(1, 8)	1.13816	1.17132	1.21347	1.23704	1.27415	1.24706	1.23207	1.26775	1.36062	1.41190
Z(1, 9)	1.08846	1.11271	1.13059	1.15124	1.17093	1.15199	1.17215	1.19867	1.24588	1.28204
Z(2, 1)	1.83434	2.09434	2.30314	3.30000	2.72751	1.83434	2.09434	2.30314	3.30000	3.06033
Z(2, 2)	1.83434	2.09434	2.30314	1.71558	1.15655	1.83434	2.09434	2.30314	1.91560	1.39049
Z(2, 3)	1.57374	1.39716	1.25169	......	......	1.80799	1.64856	1.48623	1.12267	......
Z(2, 4)	1.14572	1.01256	......	......	......	1.35107	1.15625	1.08031	......	......
Z(2, 5)	......	......	......	......	......	1.00608	......	......	......	......
Z(3, 1)	1.83434	2.09434	2.30314	1.11624	......	1.83434	2.09434	2.30314	1.23996	......
Z(3, 2)	1.49174	1.17338	1.01133	......	......	1.76077	1.35170	1.14743	......	......
Z(4, 1)	1.83434	1.31492	1.09209	......	......	1.83434	1.48671	1.16121	......	......
Z(5, 1)	1.08157	......	......	......	......	1.32553	......	......	......	......

The coupler tables are for 3,[5] 6,[6] 10, and 20 dB designs of $N=3$ to $N=21$ sections. The phase-shifter tables are for 90, 45, and 22.5 degree designs of $N=2$ to $N=9$ sections. N refers to the number of $\lambda/4$ sections in the largest cascaded coupling region of the tandem solution. It does not represent the total number of coupled $\lambda/4$ sections. The bandwidth ratios have been arbitrarily selected to be 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 14, 17, 20, and 25. The maximum normalized even-mode impedances were chosen to be 1.83434, 2.09434, 2.30314, 3.0, 3.5, and any value less than 4.5 in order have a single cascaded coupler solution. Even-mode impedances 1.83434, 2.09434, and 2.30314 represent the tightest coupling value of broadside coupled strip transmission lines for $Z_0=50\Omega$, $\epsilon_r=2.32$ (polyolefin material), and for $t=0$, $s/b=1/5$, $1/7$, $1/9$ [20], [21]. The upper limit of $Z_{oe}=4.5$ was based on the tightest coupled re-entrant configuration [16].

[5] Actually 3.0103.
[6] Actually 6.0206.

A measure of the equal-ripple performance is given as "tolerance." Although "ripple" is defined as the nominal equal-ripple value (based on Δ_{av} of Fig. 7a), the worst deviation from the coupling or phase-shift design could be $\pm$ (ripple+tolerance). For example, the $N=13$, 17:1 bandwidth -3 dB coupler, where $Z_m=1.83434$, could be as much as 0.33397 (i.e., 0.33259+.00138) dB off the 3.0103 dB design at one frequency. The tolerance of the designs given in the tables is of the order of one percent of ripple. Tolerance has the same units as ripple.

Because the phase shifters are in fact differential phase shifters, the uncoupled reference line (listed as Ref. Length) is given in terms of wavelengths at the center frequency of phase-shifter operation.[7]

Experimental Results

The basic purpose of this paper is to describe the synthesis of the components, to indicate the availability of a computer program for carrying out the synthesis, and to reference a source from which extensive tables of parameters can be obtained. In this sense, the practical realization of the component is left to the reader, as in other papers concerned with the theoretical synthesis problem.

Nevertheless, it is appropriate to make brief reference to experimental results. The components are most easily realized in a strip-transmission-line configuration consisting of three layers of dielectric with conducting strips located at the interfaces and ground planes on the outer surfaces [21]. Variation in coupling is achieved by displacing the strips along the interfaces.

Three-dB couplers and 90-degree phase shifters have been built with design bandwidth of 17:1 [22]. The couplers had 13 sections and the phase shifters 6. The polyolefin dielectric layers were 0.040, 0.020, and 0.040 inch in thickness, and the frequency range was 89 to 1500 mc/s.

The principal practical problem with these components is the reactive discontinuity at the junction between coupled sections of different coupling coefficients. It is the discontinuity which places a limit on the high-frequency operation of the components. In order to minimize these junction effects, one is forced to reduce the scale of the transmission-line cross section. The resulting smaller dimensions, however, result in increased attenuation and larger errors due to tolerances. An analysis of junction discontinuities for even and odd modes would be a valuable contribution to the further study of these components.

[7] The design tables, together with introductory test, have been deposited with ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington 25, D. C. To obtain a copy: Document 9017; advance payment $3.75 photoprints, $2.00 35mm microfilm, payable to Chief, Photoduplication Service, Library of Congress.

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